# RTT TECHNOLOGY TOPIC
# August 2011

## Transforms

A new RTT book has just been finished. This will be making it's way slowly but surely into print in a few months time and a big thank you to the many people who have contributed – too many to thank individually but you know who you are.

Having just finished typing 169,000 words of breathless prose the prospect of producing a long technology topic was significantly unappealing and anyway it is holiday time and the last thing any of you want is to read through an extended thesis on anything other than sunshine so here instead is a short story about the Olympics and the Hadamard, Fourier and Hilbert transform.

Over the past week politicians in the UK have been busy reassuring corporate sponsors and the British taxpayer that their £9.4 billion investment in the Olympics exactly twelve months away will prove to be worthwhile both in terms of immediate and longer term economic and social gain.

The longer term gain is premised on the economic transformation of the east of London, in particular Stratford and the surrounding boroughs. **The Olympic Games of 1908** based around White City to the west of central London and the **Olympic Games of 1948** also to the west of central London (Olympia and Wembley) have been cited as prior examples of money well spent though present evidence of this is rather hard to find.

The East End of London is not unfamiliar with the process of transformation. Docklands underwent a substantial and unplanned and **explosive transformation during the Second World War** but it was the invention of the container and the roll on roll off ferry in the 1950's that spelt **the end of the London docks**, ironically the decade in which the Royal Docks were the busiest that they had ever been.

The formation of the **London Development Corporation in July 1981** to an extent off set the loss of employment but the jobs that were created were different jobs requiring different people. A technology transform, the container and the roll on roll off ferry proved to be the precursor of a social and economic transform.

Rather against expectations all of the major buildings for the 2012 Olympics have been completed on time and on budget though the budget is three times larger than originally planned. The games provide substantial contractual and promotional opportunities for ICT and telecoms providers with telecoms including specialist radio, satellite links, micro wave links, mobile broadband and cable and copper and fibre upgrades.

Perhaps one of the most remarkable transformations is the increase in imaging bandwidth and the increase in addressable audience. In 1948 the BBC were an integral part of the process of audio capture and broadcast (long wave and medium and short wave radio). TV broadcasts had started again from Alexandra Palace almost exactly a year earlier which meant that the 54,000 people that owned televisions could see at least some of the action as could the 44,000 people that owned television sets in the US.

Fast forward sixty years and we find **4.7 billion people** tuned in to watch the television coverage of the 2008 Olympics in China, the first games to be filmed in high definition. It would be reasonable to expect this figure to be substantially larger for the 2012 event. This will also be the first Olympics where a majority of the participants and visitors will have smart phones suggesting that image capture will be on an altogether different scale both in terms of bandwidth and distribution.

All of this is of course made possible by three transforms, the Hadamard Transform, the Fourier Transform and the Hilbert Transform.

The mathematical basis for the **Hadamard transform** has been discussed in an earlier technology topic and the **Fourier transform** as well but as a reminder the Hadamard transform works really well with square waves, and the Fourier transform works really well with sine waves. The Fourier transform is of course used almost ubiquitously in signal processing but the Hadamard transform is pretty useful as well, for example it provides the mathematical basis for the Walsh codes used in EVDO and the OVSF codes used in the HSPA physical layer.

While Jacques Hadamard was feeding his family with elephant meat during the **Prussian siege of Paris** in 1870, an eight year old David Hilbert would have been breakfasting on something more conventional with his Prussian mum and dad.

David Hilbert was to go on to create the mathematical constructs that would allow us to represent band pass signals as complex signals which in turn would allow us to process those signals more efficiently than Nyquist had predicted. In particular it enables us to do clever things with the I and Q components that are the basis of all modern modulation techniques that in turn enable us to have smart phones and smart televisions that will allow us to watch and record the Olympics.

The clever thing about the Hilbert transform is that we can take a radio carrier which is a cosine wave and make a sine wave out of it and we can do this in both the frequency and time domain

In the frequency domain a cosine is turned into a sine by rotating the negative frequency component of the cosine by +90 degrees and the positive frequency component by – 90 degrees which we can do courtesy of creating an imaginary plane and an imaginary axis and using imaginary numbers. A sine wave put though a Hilbert transform will come out as a negative cosine, a negative cosine will come out as a negative sine wave and another transformation will bring the whole thing back to the original cosine wave though at each stage the phase will have changed by 90 degrees. This is the reason why a Hilbert transform is also called a quadrature filter, a filter that changes the phase of the spectral components depending on the sign of their frequency.

The behaviour of the Hilbert transform in the time domain is quite frankly incomprehensible at least to me but a very excellent analysis can be found **HERE**

Suffice it to see that the useful property of the transform when viewed in both the frequency and time domain is that by rotating a signal by 90 degrees it becomes orthogonal (equally distant) from the original signal which means that we can discriminate between the two modulated phase states with reference to the carrier.

If we take a pass band signal, its complex envelope spectrum is centred around zero and not the carrier frequency. The signal when represented through a Hadamard transform looks just like a one sided Fourier transform which is not surprising as this is what it is.

Nyquist states that the proper reconstruction of a baseband or pass band signal requires the signal to be sampled at twice the highest spectral frequency of the signal, for example at 200 Hz for a 100 Hz signal

The Hilbert transform allows us to use a complex envelope representation of the pass band signal to separate out the carrier leaving behind the wanted signal which is of course the base band signal. However the sampling rate is determined by the modulation rate not the highest carrier frequency. In the example referenced above the sampling frequency is therefore 6 Hz rather than 200 Hz and the computational load needed in the digital signal processor is therefore greatly reduced.

So there we have it. My Nokia C6 (yes my family have finally shamed me in to buying a smart phone) provides me with mobile broadband connectivity and image and audio capture and transmission capabilities which allow me to talk and upload and download to the internet and world wide web and so far the 1200 milliamp hour battery has not even begun to run flat though that may be because T Mobile have installed a 3G base station on the roof of the flats overlooking my house.

Even so this stuff would never have worked without that elephant, the young Prussian and a **Frenchman**

**who trained for the priesthood** but decided algebra was more interesting and useful.

The three of them and the subsequent mathematicians that followed in their footsteps produced three truly transformative transforms.

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